



# Sources of error in estimating stand transpiration using allometric relationships between stem diameter and sapwood area for *Cryptomeria japonica* and *Chamaecyparis obtusa*

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## Abstract

Estimations of stand sapwood area ( $A_{S\_stand}$ ) are among the most critical factors for determining stand transpiration ( $E$ ) using sap flux measurements.  $A_{S\_stand}$  is generally estimated using stem diameter (DBH) data measured for all trees in a stand and an allometric equation that describes the relationship between DBH and sapwood area ( $A_{S\_tree}$ ) based on available allometric data. To examine the errors in  $E$  that are generated when using the allometric equation with a small sample size, the Monte Carlo sampling technique was used with an original allometric data set. In this study, all trees (921 *Cryptomeria japonica* and 1226 *Chamaecyparis obtusa*) in the studied stands provided the original allometric data (DBH and  $A_{S\_tree}$ ). Monte Carlo simulations allow the number of sample trees required for an appropriate  $E$  estimate to be derived. The results revealed that potential errors in the  $E$  estimate were almost stable with a sample size of over 20 for both *Cr. japonica* and *Ch. obtusa*. This implies that at least, but not more than, 20 trees are necessary for the allometric equation when estimating  $E$  for both species.

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## 1. Introduction

Estimates of water fluxes in forest ecosystems can be obtained by two major approaches: eddy covariance (e.g., Kumagai et al., 2004) and sap flow (e.g., Wullschlegel et al., 2001). Eddy covariance typically

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provides a direct estimate of the water vapor flux released by the forest canopy into the atmosphere. However, this technique has many theoretical limitations, e.g., a flat and homogenous surface is required (e.g., Kaimal and Finnigan, 1994). On the other hand, the sap flow technique provides information at the single-tree scale and is not limited by complex terrains and spatial heterogeneity. This technique is especially well suited for determining the effects of species composition and other heterogeneous conditions (e.g., Wilson et al., 2001).

Despite the robustness of the sap flow technique, scaling procedures are required to extrapolate stand level estimates from sap flux density measurements of individual trees. Transpiration ( $E$ ) is scaled up to the stand level using the following equation (e.g., Granier et al., 1996b; Wilson et al., 2001; Wullschleger et al., 2001; Pataki and Oren, 2003):

$$E = J_S \frac{A_{S\_stand}}{A_G} \quad (1)$$

where  $J_S$  is the mean sap flux density,  $A_{S\_stand}$  the stand sapwood area and  $A_G$  the ground area. In short, scaling procedures need to account for two major sources of error; determinations of the total sap flow per unit stand sapwood area ( $J_S$ ) and the total sapwood area of the stand ( $A_{S\_stand}$ ).

When estimating  $J_S$ , determination of variability among trees in a stand is important because, for technical reasons, only a small number of trees can be sampled (e.g., Granier et al., 1996a). Furthermore, Hatton et al. (1995) showed that integrating radial gradients of sap flux density in the xylem (e.g., Phillips et al., 1996) for an individual tree's water-use was a major source of error in estimating stand transpiration. Wullschleger and King (2000) examined tree-to-tree variations in radial gradients of sap flux density, and calculated those that occur in the sapwood fraction involved in water transport ( $f_S$ ); this calculation is necessary for accurate extrapolation of sap flux density in individual tree to canopy scale studies (Köstner et al., 1996). According to their analysis using a Monte Carlo simulation, the coefficients of variation (CV) of the mean  $f_S$  were 7 and less than 5% in 10 and 15 sampled trees, respectively.

To estimate  $A_{S\_stand}$ , an appropriate indirect technique for measuring each tree sapwood area ( $A_{S\_tree}$ ) is required, because it is impossible to

measure the  $A_{S\_tree}$  of all trees in a stand. Conventionally,  $A_{S\_stand}$  is estimated using stem diameter (DBH) data, which is measured for all trees in a stand, and the relationship between DBH and  $A_{S\_tree}$  (e.g., Vertessy et al., 1995; Cienciala et al., 2000; Roberts et al., 2001; Wullschleger et al., 2001). The species-specific allometric equation that describes the relationship between DBH and  $A_{S\_tree}$  was developed using limited numbers of sample trees, e.g., 7–19 (Vertessy et al., 1995), 6–10 (Cienciala et al., 2000), 14–60 (Wullschleger et al., 2001) and 73 trees (Roberts et al., 2001). However, it is unclear whether an equation developed with such a limited sample size is appropriate for estimations of  $E$ , because it does not account for the scattering effect of  $A_{S\_tree}$  versus DBH. To our knowledge, no study has considered the potential errors in estimating  $E$  caused using the conventional technique for predicting  $A_{S\_stand}$ .

The primary objective of this study, therefore, was to determine how many sample trees are needed for appropriate  $E$  estimations by evaluating  $A_{S\_stand}$ . The relationships between DBH and  $A_{S\_tree}$  were obtained for all trees in the study stand and, based on the assumption that  $J_S$  was properly determined,  $E$  variability was examined by predicting  $A_{S\_stand}$  using the Monte Carlo sampling technique with the original DBH and  $A_{S\_tree}$  data set. The outcomes were used to elucidate the forest stand transpiration and water yield of the main plantation trees in Japan (*Cryptomeria japonica* and *Chamaecyparis obtuse*) (e.g., Fukuda et al., 2003).

## 2. Materials and methods

### 2.1. Site description

The experiments were conducted in even-aged 51-year-old *Cr. japonica* and *Ch. obtusa* stands in the Miyazaki Experimental Forests of Kyushu University (32°22'N, 131°09'E). The study site, which is situated in the central Kyushu mountain range at approximately 1150 m above sea level, has a mean annual temperature of 13.6 °C and mean annual precipitation of 3500 mm.

Two adjacent plantations, 2.52 and 1.74 ha plots containing 1194 *Cr. japonica* and 1426 *Ch. obtusa* trees, respectively, were used for this study. The total

stand basal area and mean DBH of the *Cr. japonica* and *Ch. obtusa* stands were 13.0 and 17.2 m<sup>2</sup> ha<sup>-1</sup>, and 27.1 and 25.8 cm, respectively. *Cr. japonica* and *Ch. obtusa* trees with a DBH of 20–30 cm represented 63 and 73% of the total stand basal area, respectively.

## 2.2. Stem diameter and estimates of sapwood area

Both study plantations were clear-cut; 921 *Cr. japonica* and 1226 *Ch. obtusa* clear-cut trees were used to develop species-specific allometric equations for estimating  $A_{S\_tree}$  from DBH. DBH and sapwood thickness (mm) were measured at approximately 2.0 m from the base of the trees at a nearby landing, and assessed as the mean of four orthogonal measurements, respectively. Color differences were used to identify the boundary between sapwood and heartwood, and in both species, this transition was easily distinguished.  $A_{S\_tree}$  was obtained by the difference between the stem cross-sectional area and heartwood area assuming that the stem cross-sections were circular.

## 2.3. Stand transpiration variability

According to Eq. (1), and assuming  $J_S$  is estimated without error, examination of errors in predicting  $A_{S\_stand}$  permits computation of  $E$  variability. Thus, we examined the potential errors in estimating  $A_{S\_stand}$  as follows.  $A_{S\_stand}$  was calculated as the sum of the  $A_{S\_tree}$  for all trees in the study stand. The  $A_{S\_tree}$  for each tree was predicted by the following allometric equation:

$$A_{S\_tree} = b \text{DBH} + a \quad (2)$$

where  $a$  and  $b$  are the fitted parameters, which are estimated from the regression intercept and slope, respectively, with the regressing  $A_{S\_tree}$  and DBH obtained from limited numbers of trees in the study stand. This equation was applied to the DBH measured for all trees in the study stands, resulting in  $A_{S\_stand}$ .

In this study, the probability density functions of  $A_{S\_stand}$  for *Cr. japonica* and *Ch. obtusa* were estimated using unstratified Monte Carlo sampling with the original data sets, namely 921 (*Cr. japonica*) and 1226 (*Ch. obtusa*)  $A_{S\_tree}$  and DBH data sets, respectively. The linear congruential method was

employed as the algorithm of random number generation. This involved random sampling of 5–100 samples from the original data pool without replacement, and applying Eq. (2), the linear regression equation for predicting the  $A_{S\_tree}$  from DBH, to each sample size ( $n$ ).  $A_{S\_stand}$  was then estimated in given sampling run. For each sample, the procedures were repeated 5000 times. Thus for a range of sample sizes, the standard deviations of  $A_{S\_stand}$  were obtained from the probability density functions, and the CV could be compared.

## 3. Results

Fig. 1 shows the relationships between DBH and  $A_{S\_tree}$  for the 921 *Cr. japonica* and 1226 *Ch. obtusa* trees. Linear regression analysis using Eq. (2) indicated that  $A_{S\_tree}$  almost correlated with DBH. The  $R^2$ -values of *Cr. japonica* and *Ch. obtusa* were 0.81 and 0.79, respectively, and even though we used curvilinear regression (Vertessy et al., 1995), these values did not improve in either species. The regression slopes for *Cr. japonica* and *Ch. obtusa* were 16.9–17.9 (95% confidence) and 11.8–12.6, respectively, and the regression intercepts were –221.0 to 180.6 and –114.6 to 96.0, respectively. In the study stands, the  $A_{S\_tree}$  for *Cr. japonica* tended to be much larger than those for *Ch. obtusa* when assuming the same DBH.

Fig. 2 compares the probability density functions of  $A_{S\_stand}/A_G$  (m<sup>2</sup> ha<sup>-1</sup>) for the given sample sizes and the  $A_{S\_stand}/A_G$  estimated from all samples. For all sample sizes, the standard deviations of the estimated  $A_{S\_stand}/A_G$  for *Cr. japonica* and *Ch. obtusa* were almost identical, although the mean  $A_{S\_stand}/A_G$  for *Ch. obtusa* was significantly larger than that for *Cr. japonica*. Fig. 3 shows the relationship between the sample size and CV for  $A_{S\_stand}/A_G$ , which was calculated using the population means and standard deviations from the above probability functions. The CV for *Cr. japonica* was larger than that for *Ch. obtusa* within the range of given sample sizes. For both trees, CV abruptly decreased with a sample size of  $n = 5$ –20 implying that the number of sample trees needed for a low CV is  $\geq 20$ . If a CV of 3% is required, sample sizes of approximately 20 and 40 trees are required for *Ch. obtusa* and *Cr. japonica*, respectively (Fig. 3).

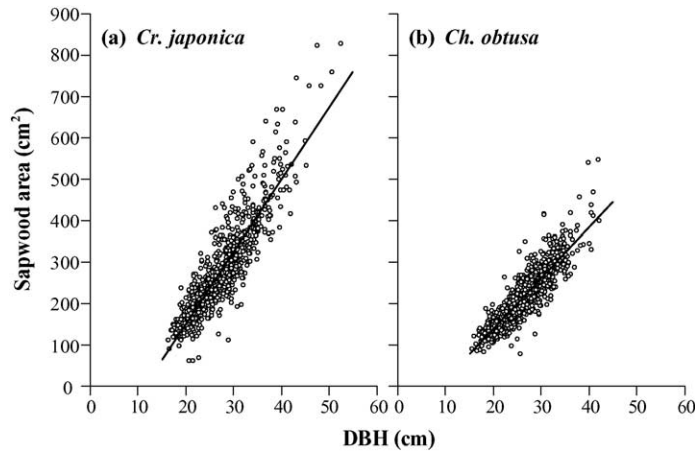


Fig. 1. Stem diameter (DBH) vs. sapwood area for the 921 *Cr. japonica* (a) and 1226 *Ch. obtusa* (b) trees in the study stands. The regression equations were  $y = 17.4x - 195.8$  ( $R^2 = 0.81$ ) and  $y = 12.2x - 105.3$  ( $R^2 = 0.79$ ) for *Cr. japonica* and *Ch. obtusa*, respectively.

#### 4. Discussion

By using the Monte Carlo sampling technique with the original allometric data (DBH and  $A_{S\_tree}$ ) of all trees in the study stands, it was possible to examine how errors in  $E$  estimates would be generated using  $A_{S\_stand}$  values estimated from measured DBH values

acquired with an allometric equation regressed with limited data. The Monte Carlo simulations permitted the number of sample trees required for appropriate  $E$  estimates to be derived. For example, the potential errors in  $E$  with a sample size of approximately  $\geq 20$  were almost stable (about one-third of the maximum CV when the sample size was 5) for both *Cr. japonica* and *Ch. obtusa*. This suggests that to make an allometric equation it might be necessary to sample at least 20 trees, but not necessarily more than 20 trees. A sample size of 5 gave a CV of less than 10% for both *Cr. japonica* and *Ch. obtusa*.

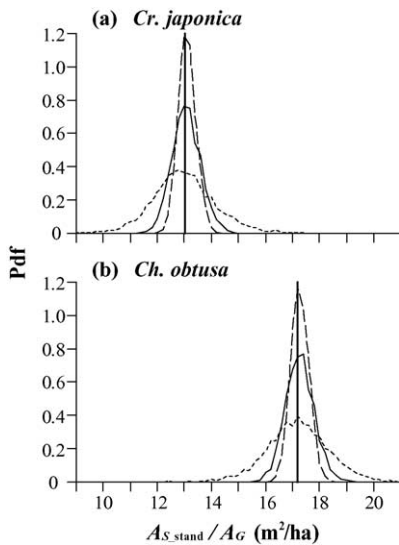


Fig. 2. Probability density function (Pdf) of estimated total stand basal area ( $A_{S\_stand}/A_G$ ) for a sample size of 5 (dotted line), 20 (solid line) and 50 (broken line). Vertical lines denote the  $A_{S\_stand}/A_G$  estimated using all samples: (a) *Cr. japonica* and (b) *Ch. obtusa* stands.

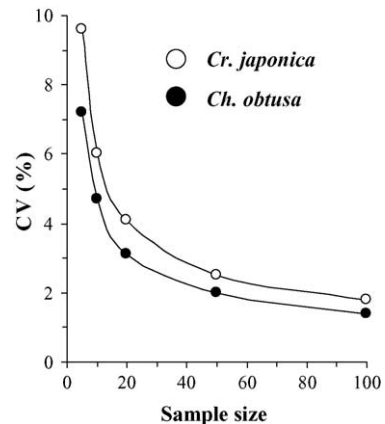


Fig. 3. Relationship between sample size and the coefficient of variation (CV) in  $A_{S\_stand}/A_G$  for *Cr. japonica* (open circle) and *Ch. obtusa* (closed circle) stands.

This study primarily focused on the source of errors in  $A_{S\_stand}$  estimates and its effect on  $E$  estimates, therefore the errors caused by mean sap flux density estimates were not accounted for. However, previous studies have shown a consistently strong linear relationship between conducting wood area and transpiration flux, suggesting that transpiration fluxes are a function of conducting wood area (Hatton et al., 1995). Vertessy et al. (1995) suggested that while there is no systematic relationship between DBH and sap flux density, single-tree transpiration is strongly related to DBH, resulting in  $A_{S\_tree}$ . Roberts et al. (2001) also pointed out that  $A_{S\_stand}$  and  $A_{S\_tree}$  are strong determinants of water-use in a forest.

Furthermore, by comparing forest stands of different ages, researchers observed that although sap flux density does not appear to be affected,  $A_S$  declines with age (Dunn and Connor, 1993; Vertessy et al., 1995; Roberts et al., 2001). As concluded by Vertessy et al. (1997), reasonable calculations of  $E$  can be made for different aged stands if a reliable relationship between age and stand sapwood area can be developed. Thus, as shown in this study, it is important to primarily consider the presence of sapwood variability as well as how such variability might affect calculations of stand transpiration.

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