

静水圧

$$\Delta p = -\rho g \Delta z$$

$$\frac{dp}{dz} = -\rho g$$

$$\int_p^{p_s} dp = -\rho g \int_{-d}^0 dz$$

$$p = \rho g d$$

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ベルヌイの定理

(1) $ma = (\rho dsA)a = \rho dsAu \frac{du}{ds}$

$$a = \frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial s} \frac{ds}{dt}$$

$$= u \frac{du}{ds}$$

(2) $F_p = -\frac{dp}{ds} dsA$

(3) $F_{gs} = -\frac{dz}{ds} \rho g dsA$

(1) = (2) + (3)

$$u \frac{du}{ds} + g \frac{dz}{ds} + \frac{1}{\rho} \frac{dp}{ds} = 0$$

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$$\frac{d}{ds} \left(\frac{u^2}{2g} + z + \frac{p}{\rho g} \right) = 0 \quad \text{where} \quad u \frac{du}{ds} = \frac{1}{2} \frac{d(u^2)}{ds}$$

$$\frac{u^2}{2g} + z + \frac{p}{\rho g} = \text{const.}$$

↓ × mg

$$\frac{1}{2} mu^2 + mgz + pV = \text{const.}$$

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ベルヌイの定理の応用

渓流流速

エネルギー損失

$$h_L \equiv f \frac{L}{D} \frac{U^2}{2g}$$

$$U = \sqrt{\frac{h_L 2gD}{L f}} \quad (1)$$

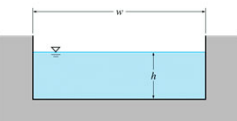
$$\frac{p_1}{\rho g} + z_1 + \frac{u_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{u_2^2}{2g} + h_L$$

$$\frac{h_L}{L} = \frac{1}{\rho g} \frac{p_1 - p_2}{L} = -\frac{1}{\rho g} \frac{dp}{dx} \quad (2)$$

(1)と(2)より
$$U = \sqrt{-\frac{2D}{\rho f} \frac{dp}{dx}} \quad (3)$$

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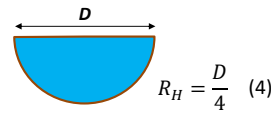
径深 R_H の定義



$$\frac{h_L}{L} \equiv S \quad (5)$$

: 損失エネルギー勾配を河床勾配と等しいと仮定

(3)、(4)、(5)より

$$R_H \equiv \frac{A}{W} = \frac{wh}{2h+w}$$


$$R_H = \frac{D}{4} \quad (4)$$

Flamantの提案 $C = \frac{R_H^{\frac{1}{6}}}{n}$ を(6)に代入

n : 粗度の関数(自然溪流で0.024~0.075)

$$U = \sqrt{S \frac{8gR_H}{f}} = \sqrt{\frac{8g}{f}} \sqrt{SR_H} = C \sqrt{SR_H} \quad (6)$$

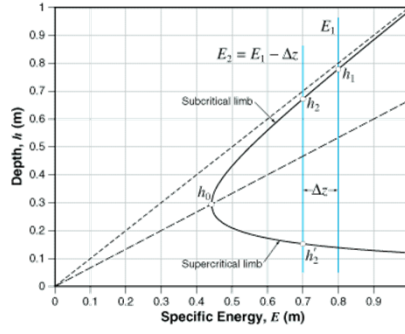
: Chézy式

$$U = \frac{R_H^{\frac{1}{6}}}{n} S^{\frac{1}{2}} R_H^{\frac{1}{2}} = \frac{1}{n} R_H^{\frac{2}{3}} S^{\frac{1}{2}} \quad (7) \quad \text{: Manning式}$$

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水位-流量関係

$E = h + \frac{u^2}{2g}$ を考える



$q = uh$ として $E = h + \frac{q^2}{2gh^2}$

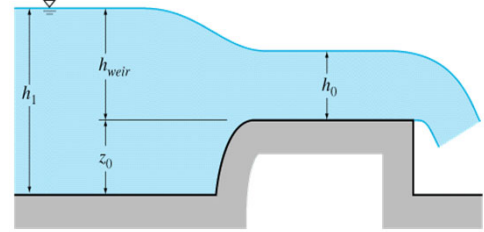
$$\left. \frac{dE}{dh} \right|_{h=h_0} = 1 - \frac{q^2}{gh_0^3} = 1 - \frac{u^2}{gh_0} = 0$$

$$\frac{u}{\sqrt{gh_0}} = F \quad \text{: フルード数 を考える}$$

$F > 1$ 射流

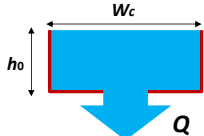
$F < 1$ 常流

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$$h_1 + \frac{u_1^2}{2g} = z_0 + h_0 + \frac{u_0^2}{2g}$$

$F = 1$ 射流-常流転換点なら流速が水位の関数になる

$$z_0 + h_w + \frac{u_1^2}{2g} = z_0 + h_0 + \frac{u_0^2}{2g}, \quad h_0 = \frac{u_0^2}{g} \quad \text{より} \quad h_w = \frac{3u_0^2}{2g}$$


$h_0 = \frac{2}{3} h_w$ であるから

$$Q = u_0 h_0 w_c = \left(\frac{8}{27g} \right)^{\frac{1}{2}} h_w^{\frac{3}{2}} w_c$$

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